

## Paper 4: Tables, Figures and Appendices

Table 1: Descriptive Statistics for Monthly Asset Returns May 1990 – July 2010

	<i>STGovBonds</i>	<i>MTGovBonds</i>	<i>LTGovBonds</i>	<i>STCorpBonds</i>	<i>MTCorpBonds</i>	<i>LTCorpBonds</i>	<i>Cash</i>	<i>Small Caps</i>	<i>Large Caps</i>	<i>Infrastructure</i>
<i>Mean</i>	0.29%	0.37%	0.51%	0.32%	0.36%	0.47%	0.13%	0.46%	0.38%	0.27%
<i>Max</i>	3.05%	5.01%	10.75%	3.46%	4.65%	14.18%	2.34%	17.24%	14.47%	4.21%
<i>Min</i>	-1.95%	-3.97%	-12.13%	-3.96%	-5.26%	-10.38%	-1.06%	-23.52%	-15.86%	-4.69%
<i>St. Deviation</i>	0.72%	1.10%	2.74%	0.84%	1.21%	2.55%	0.31%	5.68%	4.53%	1.20%
<i>Skewness</i>	0.05	-0.09	-0.35	-0.43	-0.39	0.22	1.28	-0.78	-0.61	-0.79
<i>Kurtosis</i>	3.96	4.87	5.61	6.07	5.64	9.34	13.94	5.32	4.62	9.94

Table 2: Correlations of Monthly Asset Returns May 1990 – July 2010

	<i>STGovBonds</i>	<i>MTGovBonds</i>	<i>LTGovBonds</i>	<i>STCorpBonds</i>	<i>MTCorpBonds</i>	<i>LTCorpBonds</i>	<i>Cash</i>	<i>Small Caps</i>	<i>Large Caps</i>	<i>Infrastructure</i>
<i>STGovBonds</i>	1.00	0.97	0.77	0.84	0.83	0.66	0.48	-0.15	-0.03	0.06
<i>MTGovBonds</i>		1.00	0.86	0.84	0.87	0.75	0.34	-0.12	-0.01	0.04
<i>LTGovBonds</i>			1.00	0.69	0.78	0.87	0.13	0.03	0.08	0.04
<i>STCorpBonds</i>				1.00	0.98	0.79	0.28	-0.02	0.08	0.02
<i>MTCorpBonds</i>					1.00	0.86	0.18	0.02	0.12	0.01
<i>LTCorpBonds</i>						1.00	0.07	0.15	0.21	0.01
<i>Cash</i>							1.00	-0.18	-0.07	0.22
<i>Small Caps</i>								1.00	0.79	-0.02
<i>Large Caps</i>									1.00	-0.05
<i>Infrastructure</i>										1.00

Table 3: Conditional Drawdowns of Asset Returns Full Sample

<i>alpha</i>	<i>STGovBonds</i>	<i>MTGovBonds</i>	<i>LTGovBonds</i>	<i>STCorpBonds</i>	<i>MTCorpBonds</i>	<i>LTCorpBonds</i>	<i>Cash</i>	<i>Small Caps</i>	<i>Large Caps</i>	<i>Infrastructure</i>
0.00	1.00%	1.43%	4.44%	1.03%	1.59%	3.75%	0.66%	10.57%	24.25%	0.97%
0.80	3.39%	4.31%	11.34%	3.39%	4.76%	10.88%	2.69%	33.90%	62.75%	3.12%
0.90	4.19%	5.12%	12.69%	4.29%	5.57%	12.65%	3.37%	44.06%	71.56%	3.70%
0.95	4.93%	5.67%	13.87%	4.99%	6.41%	14.11%	3.67%	54.76%	77.96%	4.23%
0.99	5.41%	6.64%	15.61%	6.11%	8.65%	19.86%	4.50%	74.61%	89.56%	5.30%

Table 4: Average Infrastructure Allocations April 1996 – July 2010

<i>alpha</i>	<i>MinCDaR</i>	<i>Portfolio 2</i>	<i>Portfolio 3</i>	<i>Portfolio 4</i>	<i>Portfolio 5</i>	<i>Portfolio 6</i>	<i>Portfolio 7</i>	<i>Portfolio 8</i>	<i>Portfolio 9</i>	<i>Portfolio 10</i>
0.00	10%	21%	33%	42%	40%	34%	23%	13%	5%	0%
0.80	9%	20%	30%	37%	40%	34%	22%	12%	5%	0%
0.90	8%	17%	27%	35%	40%	34%	23%	12%	4%	0%
0.95	9%	18%	29%	37%	39%	27%	13%	4%	1%	0%
0.99	10%	20%	28%	32%	31%	21%	12%	6%	2%	0%

*This table shows average infrastructure weights of the constructed portfolios from April 1996 – July 2010.*

Table 5: Average Constrained Infrastructure Allocations April 1996 – July 2010

<i>alpha</i>	<i>MinCDaR</i>	<i>Portfolio 2</i>	<i>Portfolio 3</i>	<i>Portfolio 4</i>	<i>Portfolio 5</i>	<i>Portfolio 6</i>	<i>Portfolio 7</i>	<i>Portfolio 8</i>	<i>Portfolio 9</i>	<i>Portfolio 10</i>
0.00	20%	19%	17%	14%	11%	10%	9%	8%	5%	0%
0.80	18%	19%	18%	15%	13%	12%	10%	8%	4%	0%
0.90	20%	20%	19%	18%	16%	14%	12%	8%	4%	0%
0.95	19%	19%	19%	19%	17%	15%	12%	8%	2%	0%
0.99	18%	19%	19%	18%	17%	14%	10%	7%	2%	0%

*This table shows average constrained infrastructure weights of the constructed portfolios from April 1996 – July 2010.*

Table 6: Relation of Monthly Asset Returns with Liabilities May 1990 – July 2010

	<i>STGovBonds</i>	<i>MTGovBonds</i>	<i>LTGovBonds</i>	<i>STCorpBonds</i>	<i>MTCorpBonds</i>	<i>LTCorpBonds</i>	<i>Cash</i>	<i>Small Caps</i>	<i>Large Caps</i>	<i>Infrastructure</i>
<i>Correlation</i>	0.56	0.58	0.58	0.49	0.52	0.52	0.25	-0.29	-0.18	0.00
<i>Beta</i>	0.11	0.17	0.42	0.11	0.17	0.35	0.02	-0.44	-0.21	0.00

Table 7: Average Infrastructure Allocations ( $\alpha = 0.95$ ) April 1996 - July 2010

<i>Target p. a.</i>	<i>MinCDaR</i>	<i>Portfolio 2</i>	<i>Portfolio 3</i>	<i>Portfolio 4</i>	<i>Portfolio 5</i>	<i>Portfolio 6</i>	<i>Portfolio 7</i>	<i>Portfolio 8</i>	<i>Portfolio 9</i>	<i>Portfolio 10</i>
2.00%	30%	37%	42%	43%	33%	20%	9%	3%	1%	0%
3.00%	44%	45%	46%	40%	28%	16%	8%	2%	0%	0%
4.00%	49%	49%	43%	33%	23%	13%	5%	1%	0%	0%

*This table shows average infrastructure weights of the constructed portfolios from April 1996 – July 2010 for different target rates.*

Table 8: Average Downside Betas with the S&amp;P 500 May 1990 – July 2010

<i>STGovBonds</i>	<i>MTGovBonds</i>	<i>LTGovBonds</i>	<i>STCorpBonds</i>	<i>MTCorpBonds</i>	<i>LTCorpBonds</i>	<i>Cash</i>	<i>Small Caps</i>	<i>Large Caps</i>	<i>Infrastructure</i>
0.05	0.11	0.36	0.06	0.13	0.31	-0.01	1.27	0.99	-0.07

Table 9: Average Downside Beta Hedged Infrastructure Allocations April 1996 – July 2010

Downside Beta Restriction = 0.4										
<i>alpha</i>	<i>MinCDaR</i>	<i>Portfolio 2</i>	<i>Portfolio 3</i>	<i>Portfolio 4</i>	<i>Portfolio 5</i>	<i>Portfolio 6</i>	<i>Portfolio 7</i>	<i>Portfolio 8</i>	<i>Portfolio 9</i>	<i>Portfolio 10</i>
0.00	10%	19%	26%	34%	38%	34%	30%	25%	21%	16%
0.80	9%	17%	25%	31%	36%	36%	29%	23%	19%	16%
0.90	8%	15%	21%	29%	36%	37%	30%	23%	18%	16%
0.95	9%	15%	23%	30%	37%	39%	32%	23%	15%	16%
0.99	10%	17%	23%	30%	34%	33%	29%	21%	14%	16%
Downside Beta Restriction = 0.2										
<i>alpha</i>	<i>MinCDaR</i>	<i>Portfolio 2</i>	<i>Portfolio 3</i>	<i>Portfolio 4</i>	<i>Portfolio 5</i>	<i>Portfolio 6</i>	<i>Portfolio 7</i>	<i>Portfolio 8</i>	<i>Portfolio 9</i>	<i>Portfolio 10</i>
0.00	10%	17%	24%	31%	34%	30%	28%	27%	24%	22%
0.80	9%	16%	23%	29%	32%	33%	29%	26%	24%	22%
0.90	8%	14%	19%	26%	32%	35%	31%	26%	24%	22%
0.95	9%	14%	20%	27%	33%	36%	32%	28%	25%	22%
0.99	10%	16%	21%	27%	31%	32%	31%	29%	26%	22%

This table shows average infrastructure weights of the downside beta hedged portfolios from April 1996 – July 2010.

Table 10: Average Conditional Drawdowns of Portfolio Returns with and without Infrastructure April 1996 - July 2010

<i>alpha</i>	<i>MinCDaR</i>	<i>Portfolio 2</i>	<i>Portfolio 3</i>	<i>Portfolio 4</i>	<i>Portfolio 5</i>	<i>Portfolio 6</i>	<i>Portfolio 7</i>	<i>Portfolio 8</i>	<i>Portfolio 9</i>	<i>Portfolio 10</i>
0.00	0.11%	0.18%	0.31%	0.47%	0.65%	0.91%	1.28%	1.75%	2.43%	4.89%
	0.11%	0.10%	0.17%	0.26%	0.44%	0.74%	1.17%	1.68%	2.40%	4.89%
0.80	0.44%	0.77%	1.29%	1.85%	2.49%	3.32%	4.44%	5.81%	8.04%	15.97%
	0.44%	0.48%	0.76%	1.12%	1.74%	2.73%	4.03%	5.56%	7.94%	15.97%
0.90	0.60%	0.99%	1.64%	2.36%	3.16%	4.22%	5.63%	7.33%	10.35%	20.59%
	0.60%	0.65%	1.00%	1.44%	2.26%	3.57%	5.26%	7.17%	10.29%	20.59%
0.95	0.70%	1.18%	1.88%	2.68%	3.60%	4.80%	6.44%	8.47%	12.20%	24.00%
	0.70%	0.78%	1.15%	1.66%	2.73%	4.31%	6.25%	8.41%	12.18%	24.00%
0.99	0.86%	1.37%	2.10%	2.96%	3.95%	5.32%	7.22%	9.63%	14.48%	27.75%
	0.86%	1.01%	1.37%	1.98%	3.11%	4.75%	6.91%	9.50%	14.45%	27.75%

Figures denote the averages of the conditional portfolio drawdowns with and without infrastructure. For each alpha, the first row denotes the average conditional portfolio drawdown when the portfolio is constructed without infrastructure. The second row shows portfolio risk with infrastructure for the **identical expected portfolio return**.

Figure 1

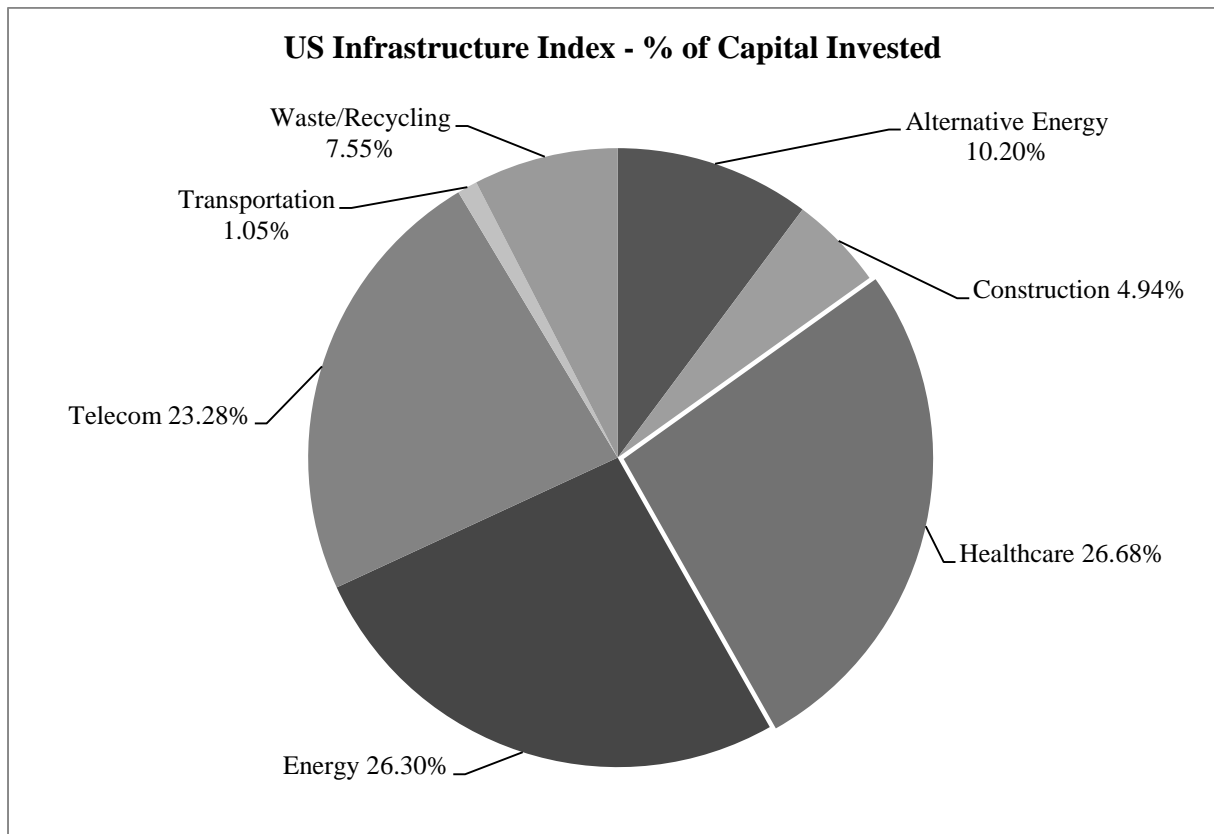


Figure 2: Time-Varying Infrastructure Allocations ( $\alpha = 0.95$ )

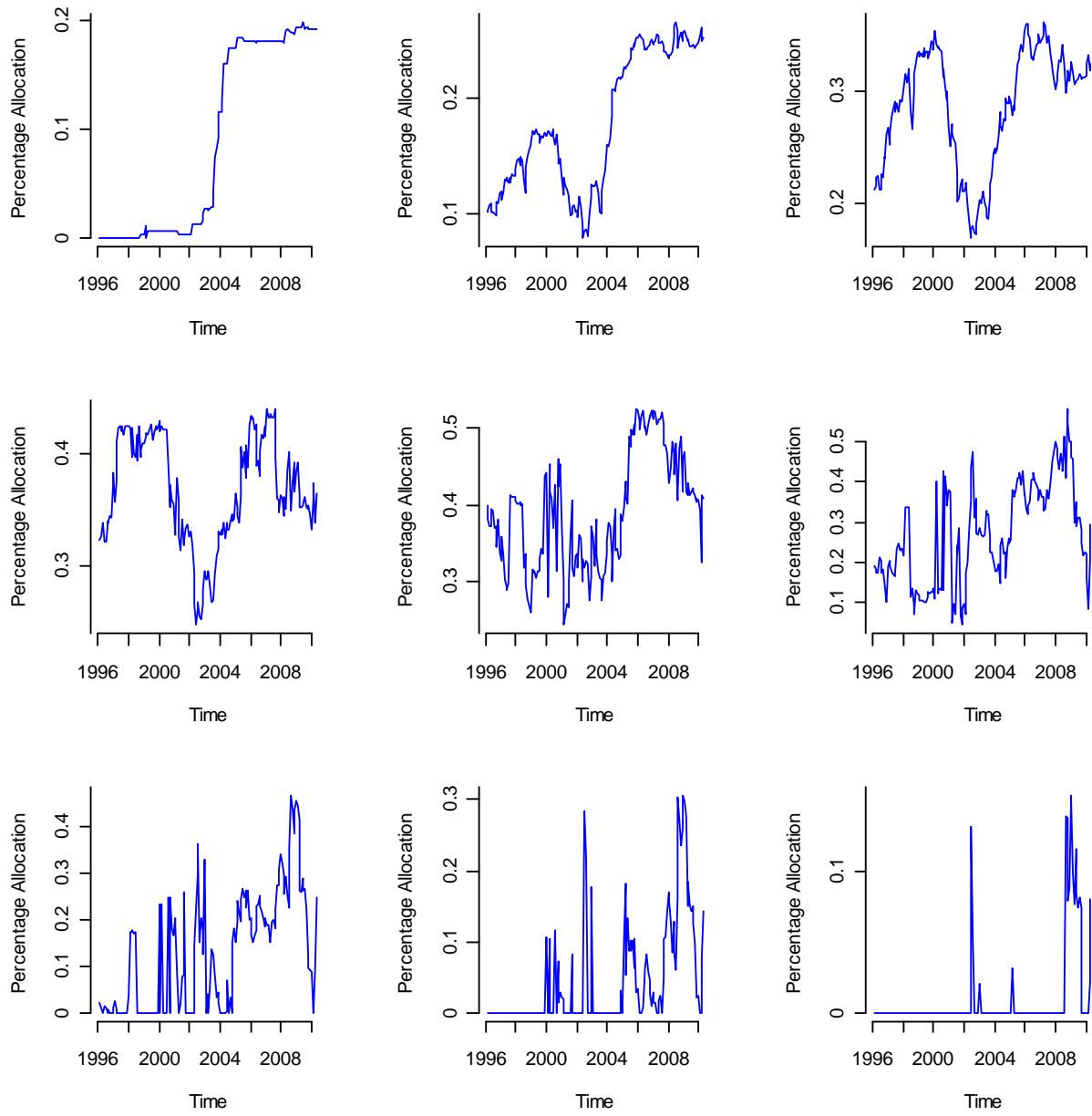


Figure 3: Time-Varying Infrastructure Allocations ( $\alpha = 0.00$ )

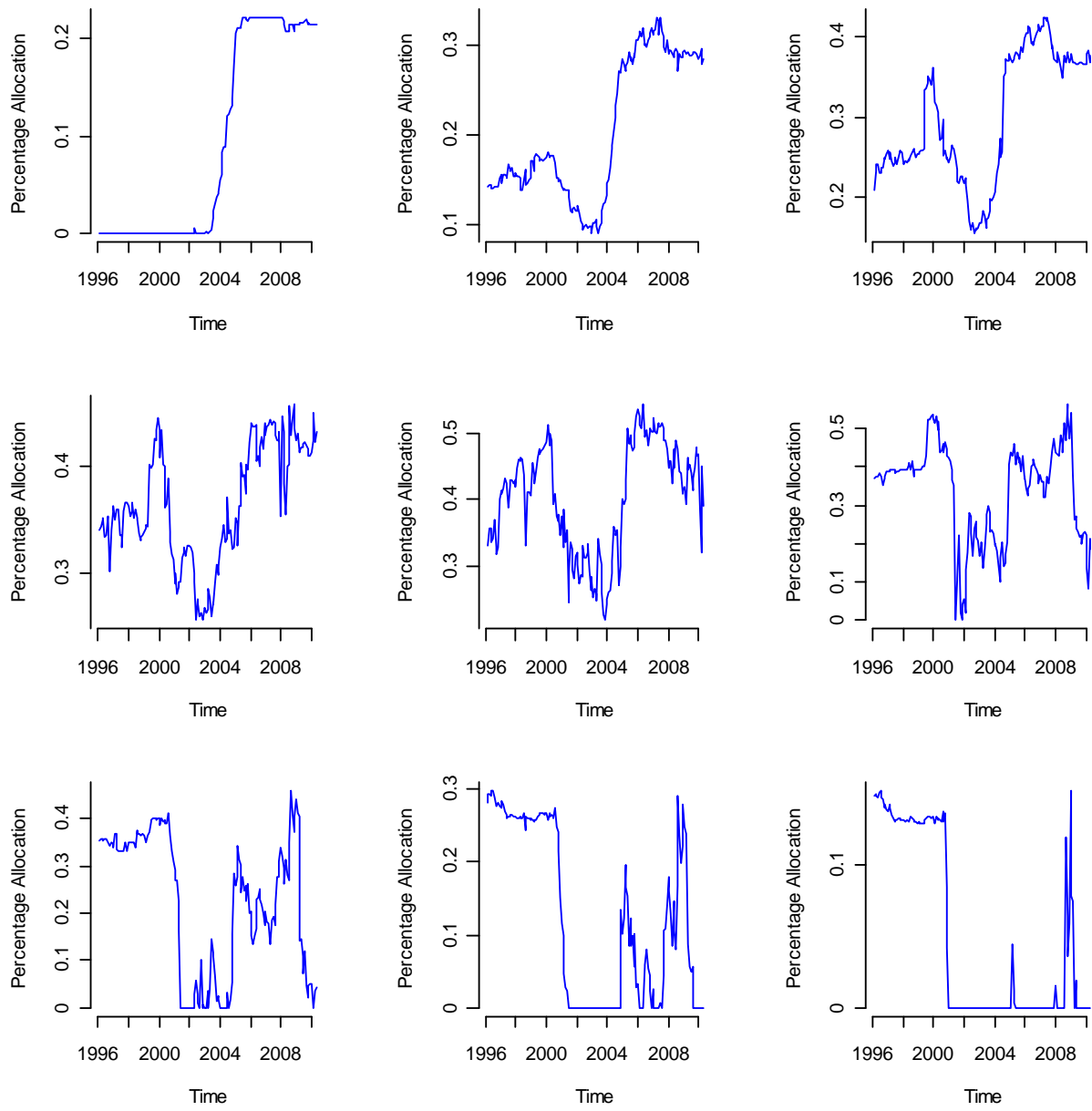


Figure 4: Time-Varying Infrastructure Allocations ( $\alpha = 0.80$ )

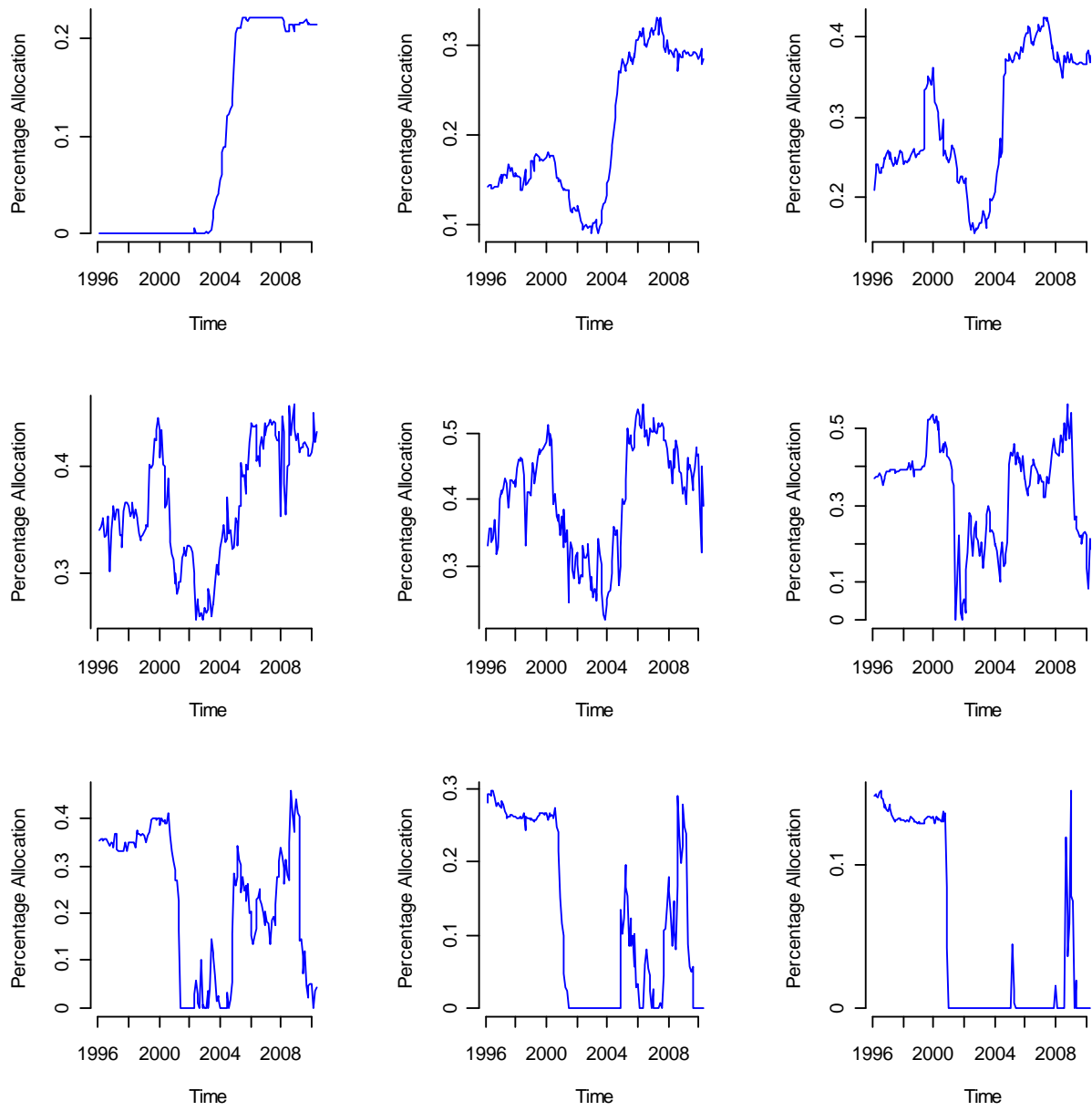


Figure 5: Time-Varying Infrastructure Allocations ( $\alpha = 0.90$ )

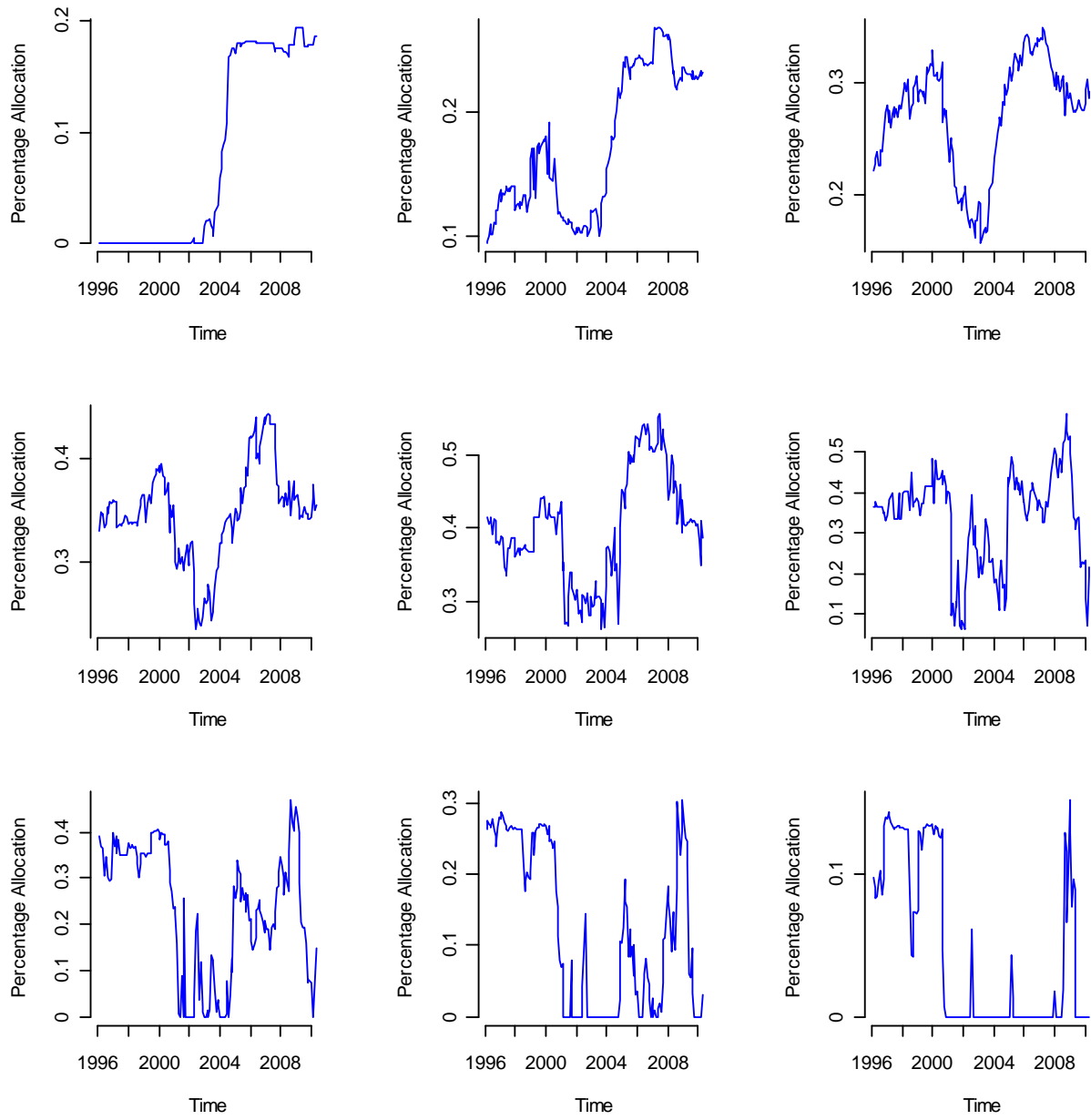




Figure 6: Time-Varying Infrastructure Allocations ( $\alpha = 0.99$ )

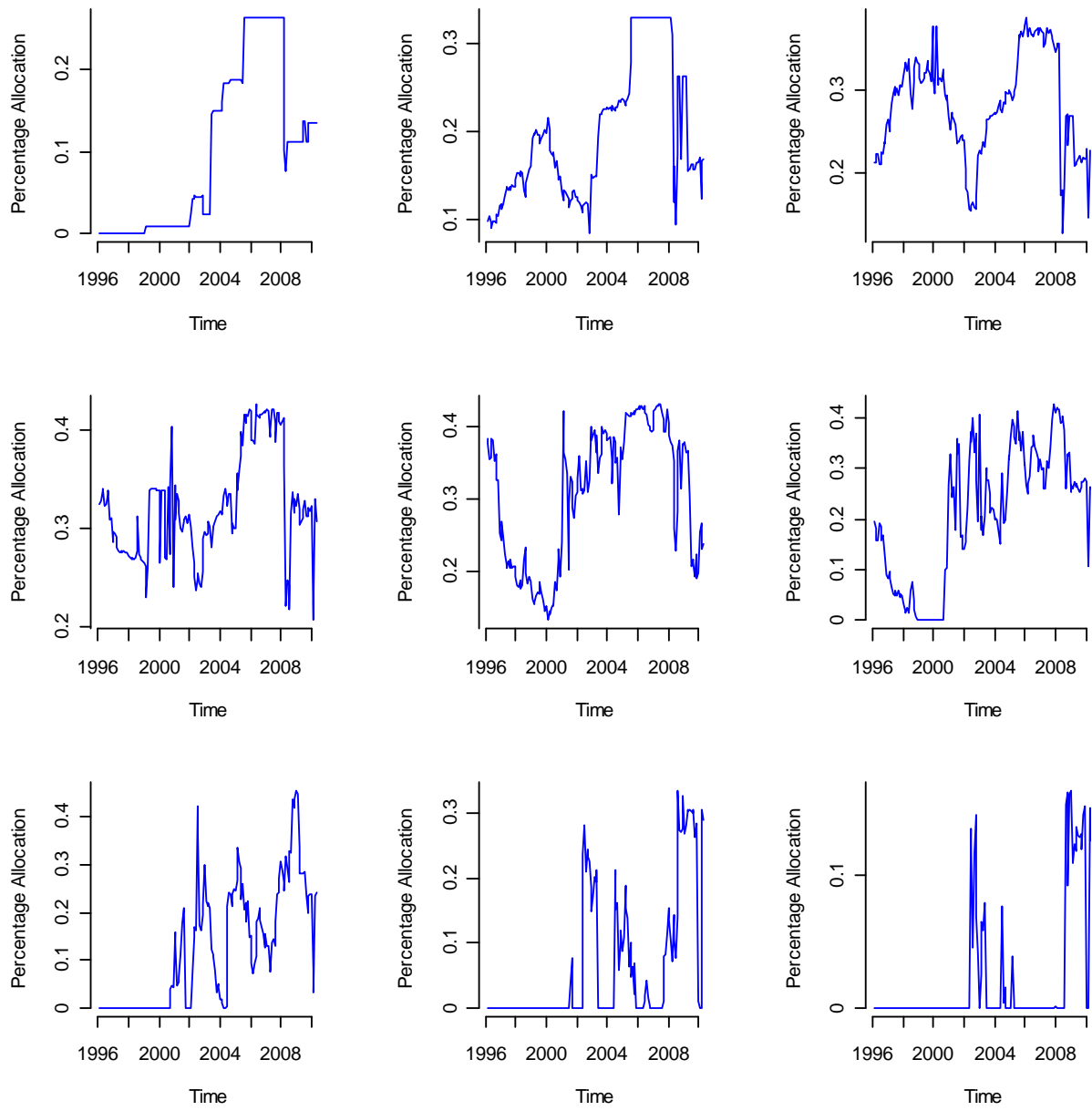


Figure 7: Time-Varying Infrastructure Allocations ( $\alpha = 0.95$ ) Target = 2

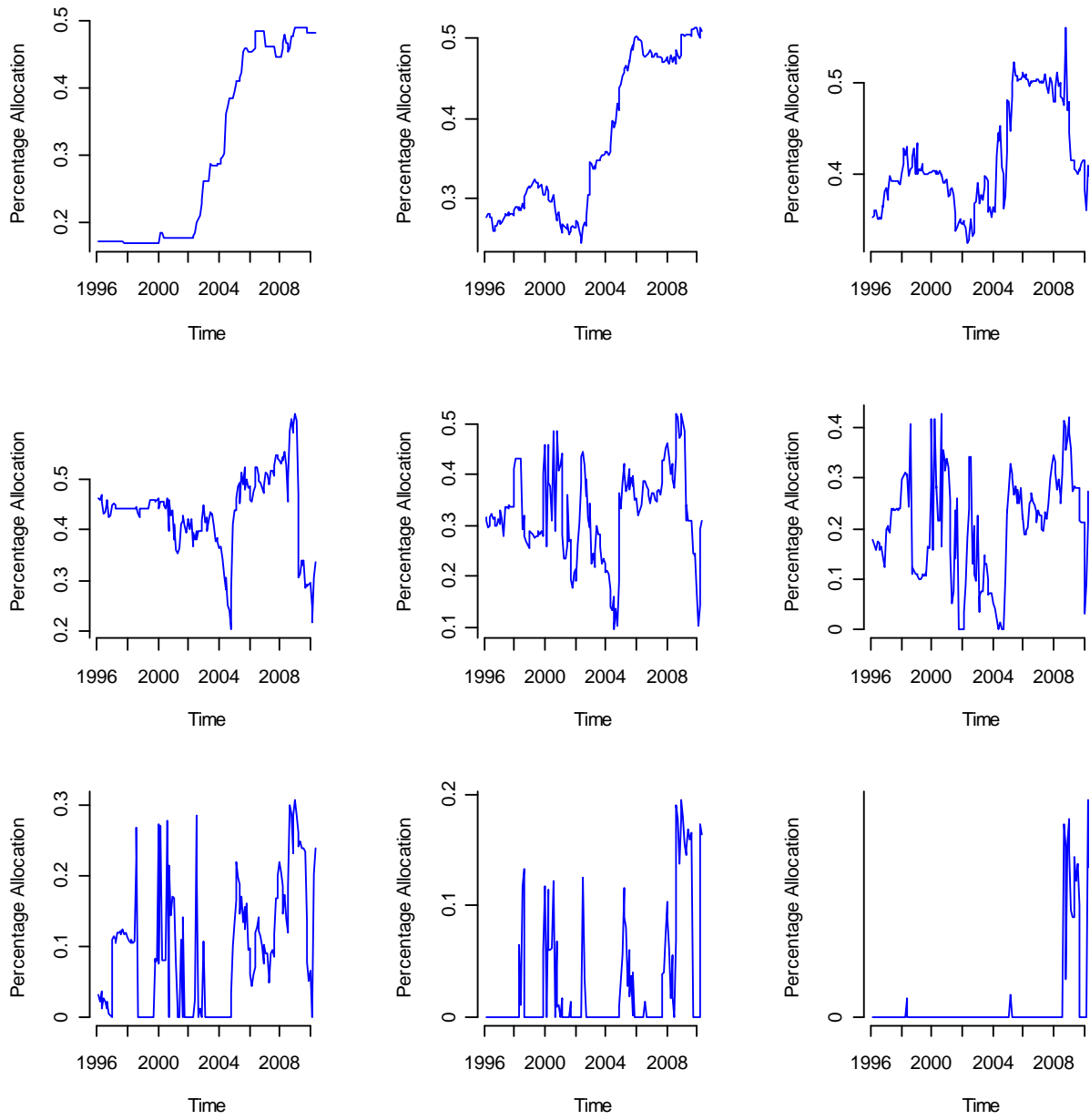


Figure 8: Time-Varying Infrastructure Allocations ( $\alpha = 0.95$ ) Target = 3

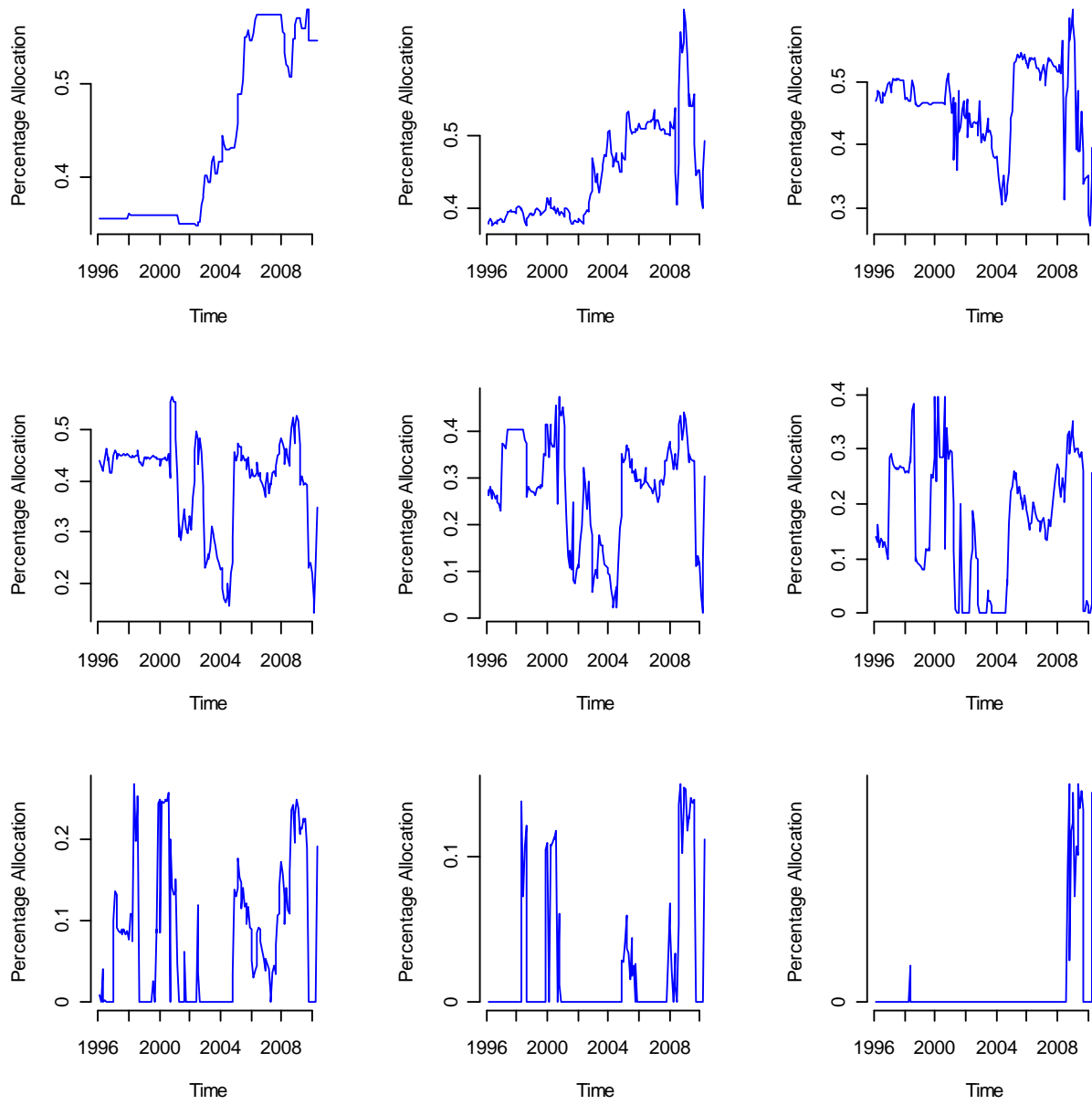


Figure 9: Time-Varying Infrastructure Allocations ( $\alpha = 0.95$ ) Target = 4

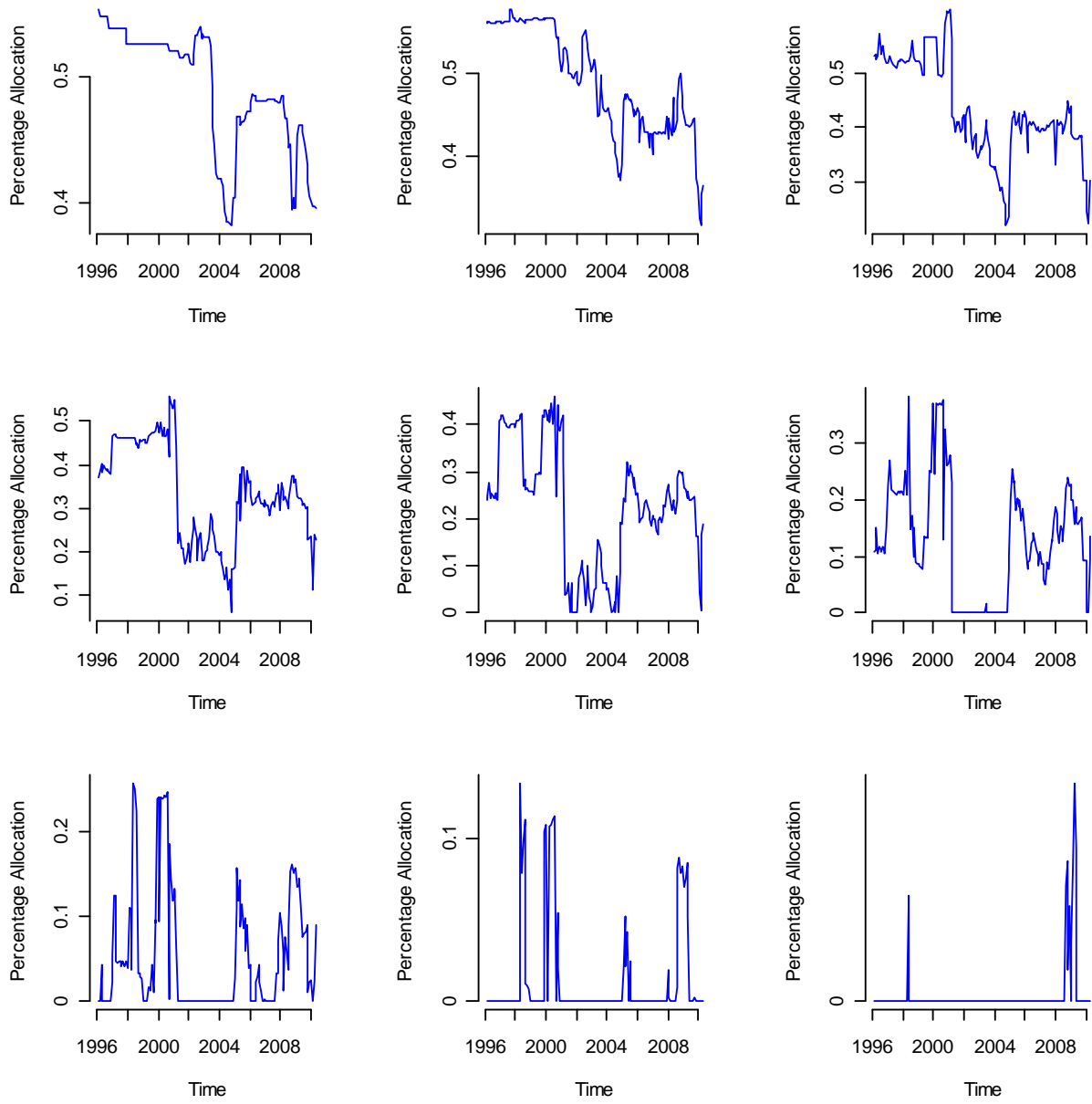


Figure 10: Time-Varying Infrastructure Allocations ( $\alpha = 0.95$ ) Downside Beta Restriction= 0.4

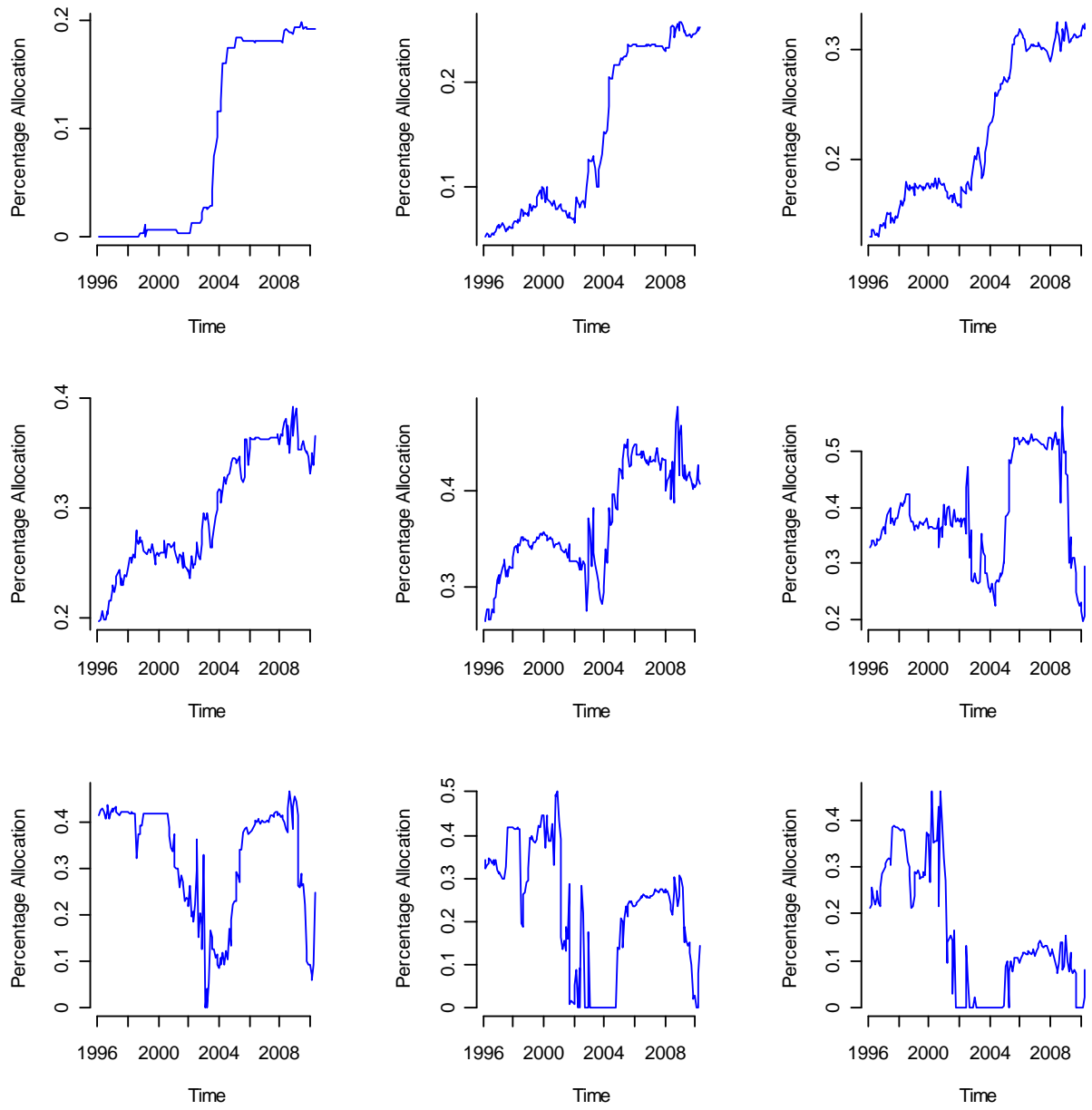
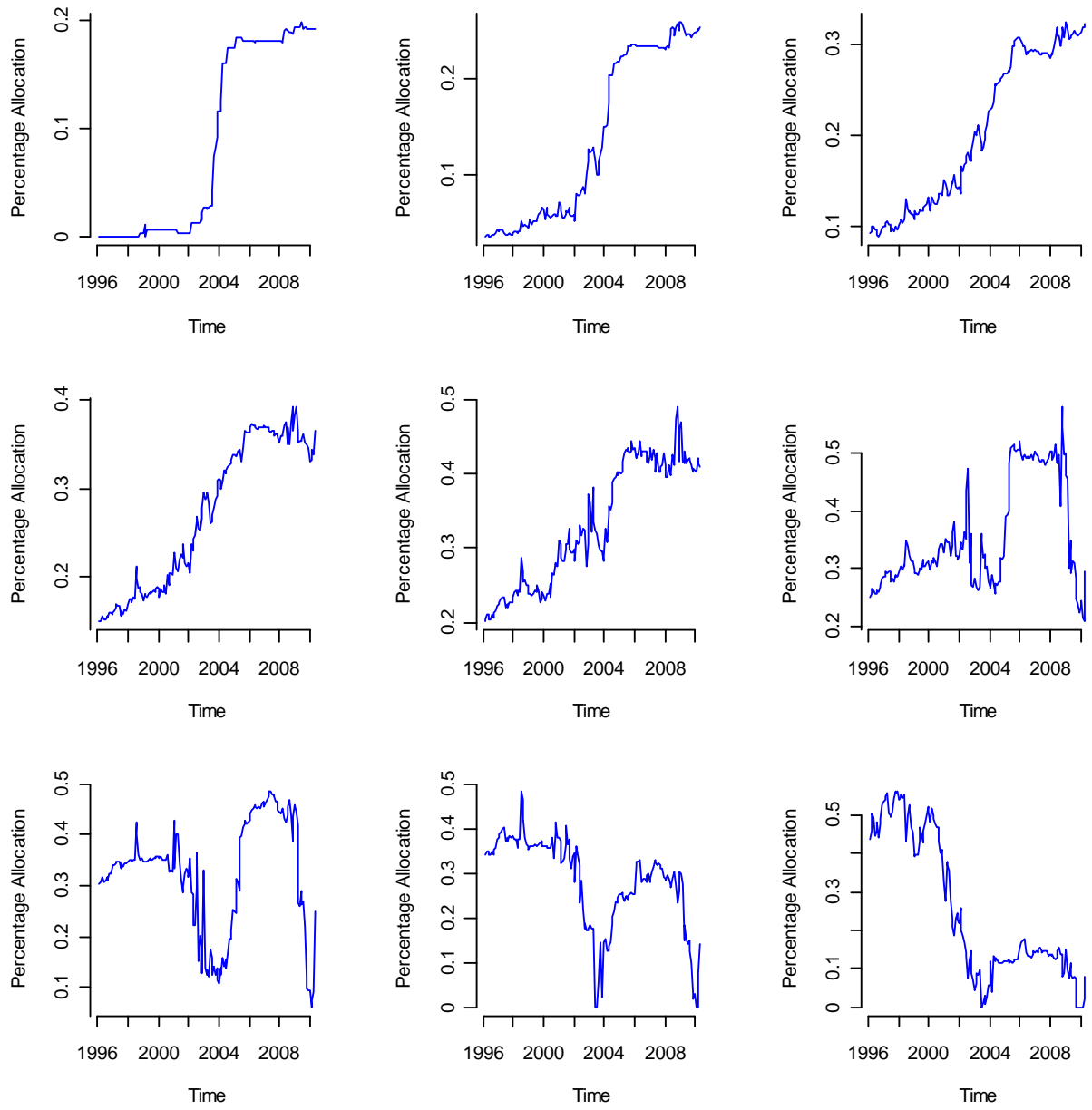


Figure 11: Time-Varying Infrastructure Allocations ( $\alpha = 0.95$ ) Downside Beta Restriction = 0.2



#### Paper 4: Appendix

The optimization problem in which drawdown risk is minimized for a certain level of expected return at each point in time is reduced to the following linear programming problem.

*Minimize* (in  $x, u, z, \eta$ )  $z$

*s.t.*

$$\eta + \frac{1}{(1-\alpha)J} \sum_{k=1}^J z_k \leq z$$

$$z_k \geq u_k - y_k * x - \eta \quad 1 \leq k \leq J$$

$$z_k \geq 0 \quad 1 \leq k \leq J$$

$$u_k \geq y_k * x \quad 1 \leq k \leq J$$

$$u_k \geq u_{k-1} \quad 1 \leq k \leq J$$

$$u_0 = 0$$

$$R(x) = \kappa$$

$$0 \leq x_i \leq 1$$

$$\sum_{i=1}^n x_i = 1,$$

where  $y_k$  is a vector of cumulative asset returns up to the time moment  $k$ ,  $x$  is a vector of portfolio weights, and  $z_k$  as well as  $u_k$ ,  $1 \leq k \leq J$  are auxiliary variables.  $\kappa$  is a predetermined value for the return on the portfolio  $R(x)$ .

If a constraint on downside beta is imposed, the optimization problem at each point in time is:

*Minimize* (in  $x, u, z, \eta$ )  $z$

*s.t.*

$$\eta + \frac{1}{(1-\alpha)J} \sum_{k=1}^J z_k \leq z$$

$$z_k \geq u_k - y_k * x - \eta \quad 1 \leq k \leq J$$

$$z_k \geq 0 \quad 1 \leq k \leq J$$

$$u_k \geq y_k * x \quad 1 \leq k \leq J$$

$$u_k \geq u_{k-1} \quad 1 \leq k \leq J$$

$$u_0 = 0$$

$$\beta_p^- \leq d$$

$$R(x) = \kappa$$

$$0 \leq x_i \leq 1$$

$$\sum_{i=1}^n x_i = 1,$$

Where  $d$  is the maximum value allowed for portfolio downside beta.